Baryon resonance EM transition form factors at high Q^2 in a light-cone quark model

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- Calculations of EM transition form factors from N to N* with Brad Keister, NSF
  - Light-cone (relativistic) quark model fit to nucleon elastic form factors
  - Baryon wave functions found by solving a three-quark Hamiltonian
  - Calculate strong-decay signs using pair-creation ($^3P_0$) model

EM transition form factors

- Rigorous approaches underway:
  - Schwinger-Dyson Bethe-Salpeter studies
  - Lattice QCD
- Quark-model calculations
  - Most reliable use light-front dynamics to improve one-body current approximation N, Δ, Roper, N*(1535):
    - Terent'ev, Weber, Dziembowski, Chung & Coester, Schlumpf, Aznauryan, Rome group, Miller
    - Relativistic effects are large
      - Need to remove interaction dependence of boosts
      - Minimize effect of ignored two-body currents
    - Other groups use point-form

Light-cone model of EM form factors

- Construct baryon wave functions in baryon CM frame in terms of free-particle light-front spinors
  - Bakamjian-Thomas construction
- Evaluate matrix elements of one-body EM current using these wave functions
- Find helicity amplitudes for EM transitions in terms of reduced matrix elements

Light-front dynamics

- Light-front Hamiltonian dynamics
  - Constituents are treated as particles rather than fields
  - Certain combinations of boosts and rotations are independent of the interactions which govern quark dynamics
    - Simplifies calculations of matrix elements in which composite baryons recoil with large momenta
  - Use complete orthonormal set of basis states
    - Composed of three constituent quarks
    - Satisfy rotational covariance

Calculation scheme

- Bakamjian and Thomas scheme:
  - Three-body relativistic bound-state problem is solved for the wave functions of baryons with the assumption of three interacting constituent quarks
  - Wave functions used to calculate the matrix elements of one (and in principle, two, and three)-body electromagnetic current operators

Calculational details

- Expand in sets of free-particle states:
  - Evaluate I^+ (EM) current matrix element by expanding baryon wave function in terms of light-front spinors for the quarks

$$\langle M' j; \tilde{\mathbf{P}}' \mu' | I^+(0) | M j; \tilde{\mathbf{P}} \mu \rangle =$$

$$(2\pi)^{-18} \int d\tilde{\mathbf{p}}_1' \int d\tilde{\mathbf{p}}_2' \int d\tilde{\mathbf{p}}_3' \int d\tilde{\mathbf{p}}_1 \int d\tilde{\mathbf{p}}_2 \int d\tilde{\mathbf{p}}_3 \sum \langle M' j'; \tilde{\mathbf{P}}' \mu' | \tilde{\mathbf{p}}_1' \mu_1' \tilde{\mathbf{p}}_2' \mu_2' \tilde{\mathbf{p}}_3' \mu_3' \rangle$$

$$\times \langle \tilde{\mathbf{p}}_1' \mu_1' \tilde{\mathbf{p}}_2' \mu_2' \tilde{\mathbf{p}}_3' \mu_3' | I^+(0) | \tilde{\mathbf{p}}_1 \mu_1 \tilde{\mathbf{p}}_2 \mu_2 \tilde{\mathbf{p}}_3 \mu_3 \rangle \langle \tilde{\mathbf{p}}_1 \mu_1 \tilde{\mathbf{p}}_2 \mu_2 \tilde{\mathbf{p}}_3 \mu_3 | M j; \tilde{\mathbf{P}} \mu \rangle.$$

  - Need baryon state vectors written in terms of wave functions

Calculational details…

- Expand in sets of free-particle states:

$$\langle \tilde{\mathbf{p}}_1\mu_1\tilde{\mathbf{p}}_2\mu_2\tilde{\mathbf{p}}_3\mu_3|Mj;\tilde{\mathbf{P}}\mu\rangle =$$

$$\left|\frac{\partial(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{p}}_3)}{\partial(\tilde{\mathbf{P}},\mathbf{k}_1,\mathbf{k}_2)}\right|^{-1/2}(2\pi)^3\delta(\tilde{\mathbf{p}}_1+\tilde{\mathbf{p}}_2+\tilde{\mathbf{p}}_3-\tilde{\mathbf{P}})\langle\tfrac{1}{2}\bar{\mu}_1\tfrac{1}{2}\bar{\mu}_2|s_{12}\mu_{12}\rangle\langle s_{12}\mu_{12}\tfrac{1}{2}\bar{\mu}_3|s\mu_s\rangle$$
$$\times\langle l_\rho\mu_\rho l_\lambda\mu_\lambda|L\mu_L\rangle\langle L\mu_L s\mu_s|j\mu\rangle Y_{l_\rho\mu_\rho}(\hat{\mathbf{k}}_\rho)Y_{l_\lambda\mu_\lambda}(\hat{\mathbf{K}}_\lambda)\Phi(k_\rho,K_\lambda)$$
$$\times D^{(1/2)\dagger}_{\bar{\mu}_1\mu_1}[\underline{R}_{cf}(k_1)]D^{(1/2)\dagger}_{\bar{\mu}_2\mu_2}[\underline{R}_{cf}(k_2)]D^{(1/2)\dagger}_{\bar{\mu}_3\mu_3}[\underline{R}_{cf}(k_3)].$$

Calculational details…

- Cluster expansion of electromagnetic current operator $$I^\mu(x) = \sum_j I_j^\mu(x) + \sum_{j<k} I_{jk}^\mu(x) + \cdots .$$
- We evaluate only one-body matrix elements and assume struck quark has EM current of free Dirac particle

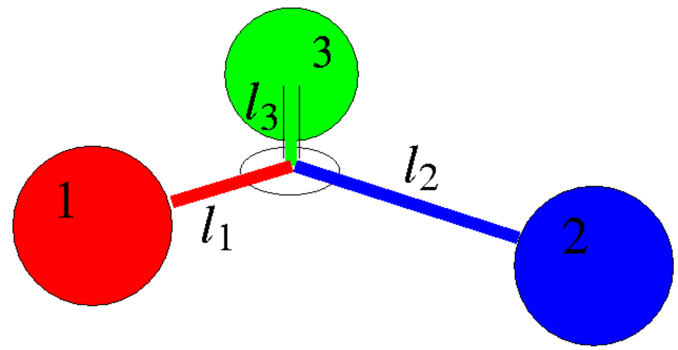
$$\langle \tilde{\mathbf{p}}'\mu' | I^+(0) | \tilde{\mathbf{p}}\mu \rangle = F_{1q}(Q^2)\delta_{\mu'\mu} - i(\sigma_y)_{\mu'\mu}\frac{Q}{2m_i}F_{2q}(Q^2)$$

  - Result is a 6D integral that we evaluate using numerical techniques [quasi-random number (Sobol) sequences]

Light-cone model…

- Wave functions expanded in h.o. basis up to N=6 or 7 ($\hbar\omega$)
  - e.g. 50 components for N and Roper, 70 for N(1535)S_{11}
- Requires simultaneous calculation of strong-decay amplitudes
  - Calculate Nπ sign using $^3P_0$ model using *identical* wave functions
- Fit quark EM form factors to nucleon EM form factors (moments and Q^2 dependence)
  - Similar calculations performed by Rome group (Cardarelli, Pace, Salme, Simula)

Model of spectrum and wave functions



- Confinement:
  - Flux tubes, combined with adiabatic approx.
  - minimum length string:
    $V_B(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=\sigma(l_1+l_2+l_3)=\sigma L_{min}$
  - linear at large q-junction separations

- Short-range interactions:
  - Ground-state spectrum suggests flavor-dependent short-range (contact) interactions
  - Use OGE (other possibilities: OBE, instanton-induced interactions)

Wave functions

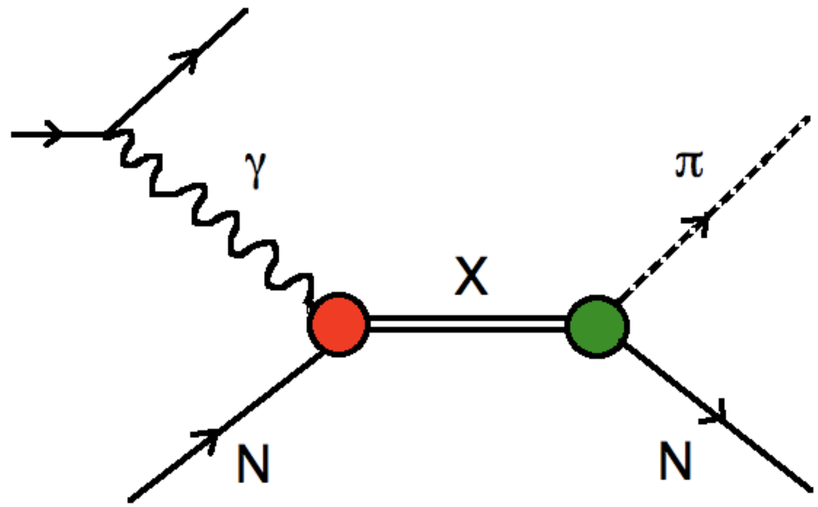
- Variational calculation in large HO basis (SC, N. Isgur)
  - String confinement, plus associated spin-orbit
  - Include OGE Coulomb, contact, tensor, spin-orbit
  - Relativistic KE, relativistic corrections in potentials, e.g.

$$\left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\rm cont}} \frac{8\pi}{3}\alpha_s(r_{ij})\frac{2}{3}\frac{\mathbf{S}_i\cdot\mathbf{S}_j}{m_i m_j}\left[\frac{\sigma_{ij}^3}{\pi^{\frac{3}{2}}}e^{-\sigma_{ij}^2 r_{ij}^2}\right]\left(\frac{m_i m_j}{E_i E_j}\right)^{\frac{1}{2}+\epsilon_{\rm cont}}$$

  - Contact interaction smeared with Gaussian form factor, σ_{ij} depends on quark flavor (1.8 GeV for light quarks)

Electro/photo-production amplitude signs

- Experiments measure interference of products of amplitudes $A^{\dagger}_{X\text{-}\gamma N}\, A_{X\text{-}N\pi}$ with nucleon Born term and/or each other
- Phase of either depends on sign conventions in N and X wave fns
- Phase of product does not!

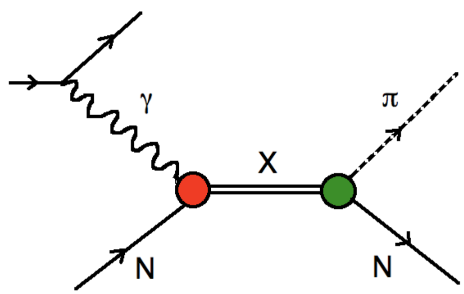


Electro/photo-production amplitude signs…

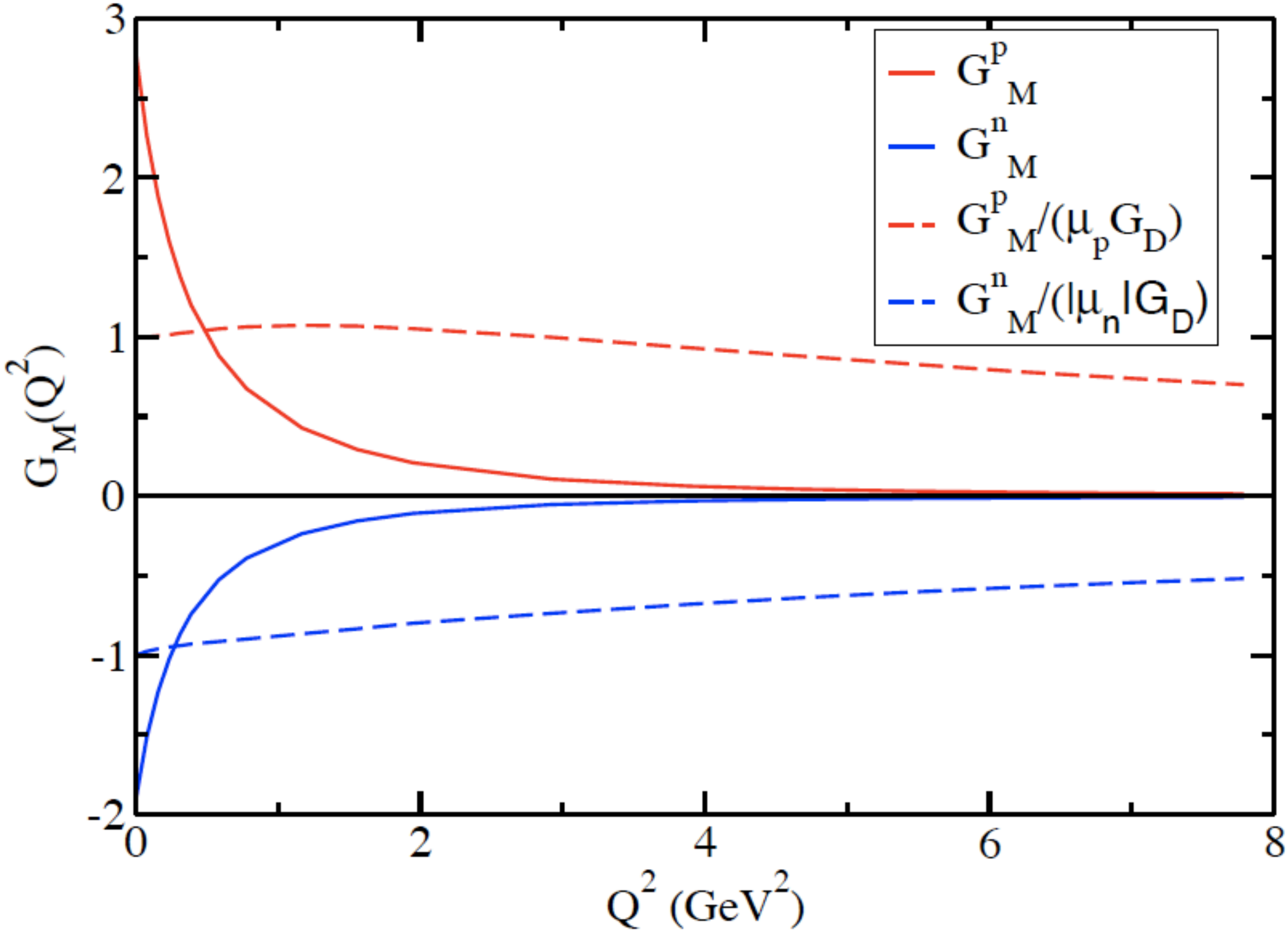
- Photo- and electro-production amplitudes quoted in analyses are the products

  $$A^{\dagger}_{X\text{-}\gamma N}\, A_{X\text{-}N\pi} \,/\, |A_{X\text{-}N\pi}|$$

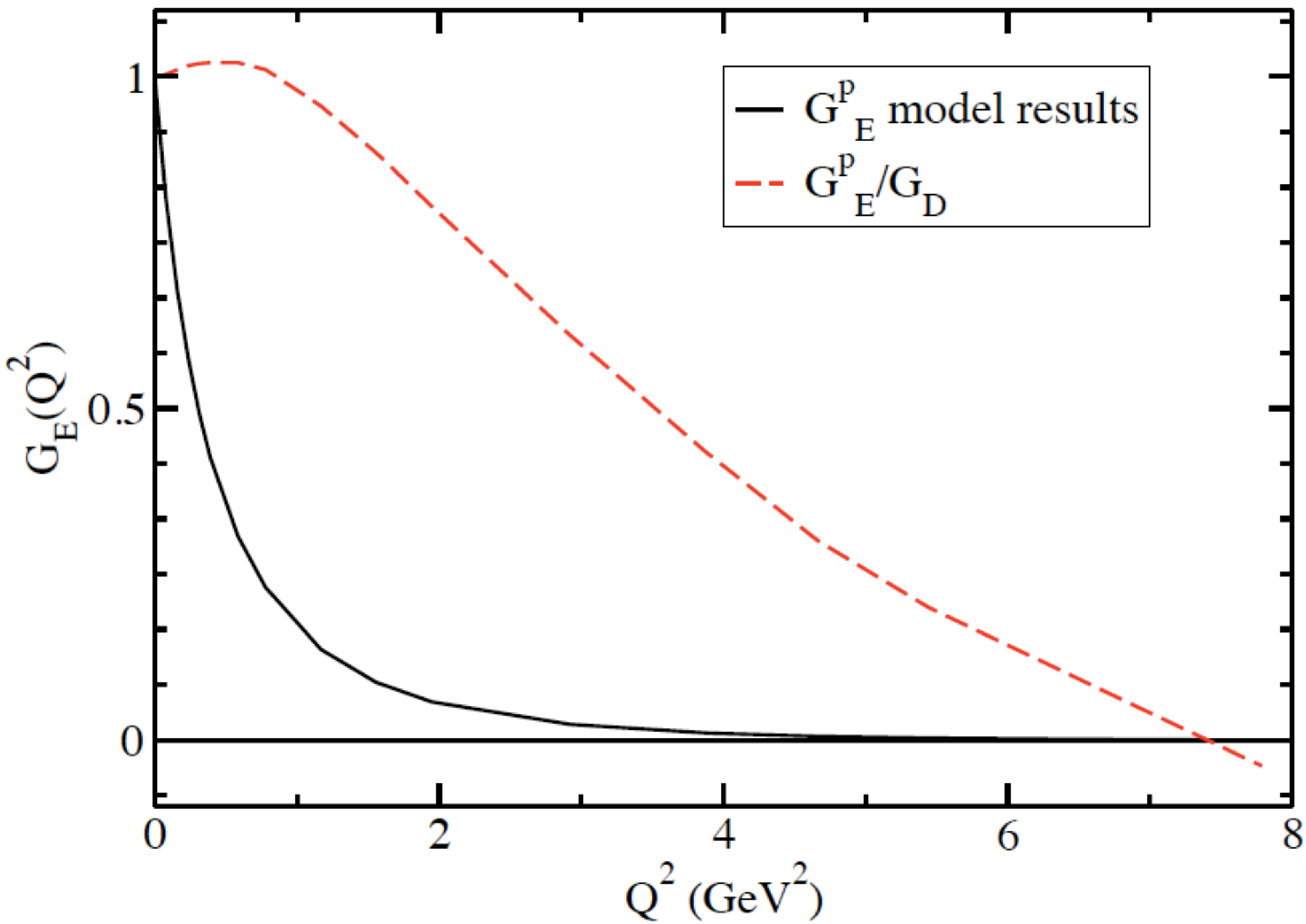
  - Phase of $A_{X\text{-}N\pi}$ not measurable in $N\pi$ elastic scattering
  - Theorists must calculate $A_{X\text{-}N\pi}$ with exactly the same X and N wave functions used to calculate $A_{X\text{-}\gamma N}$
  - We use $^3P_0$ model



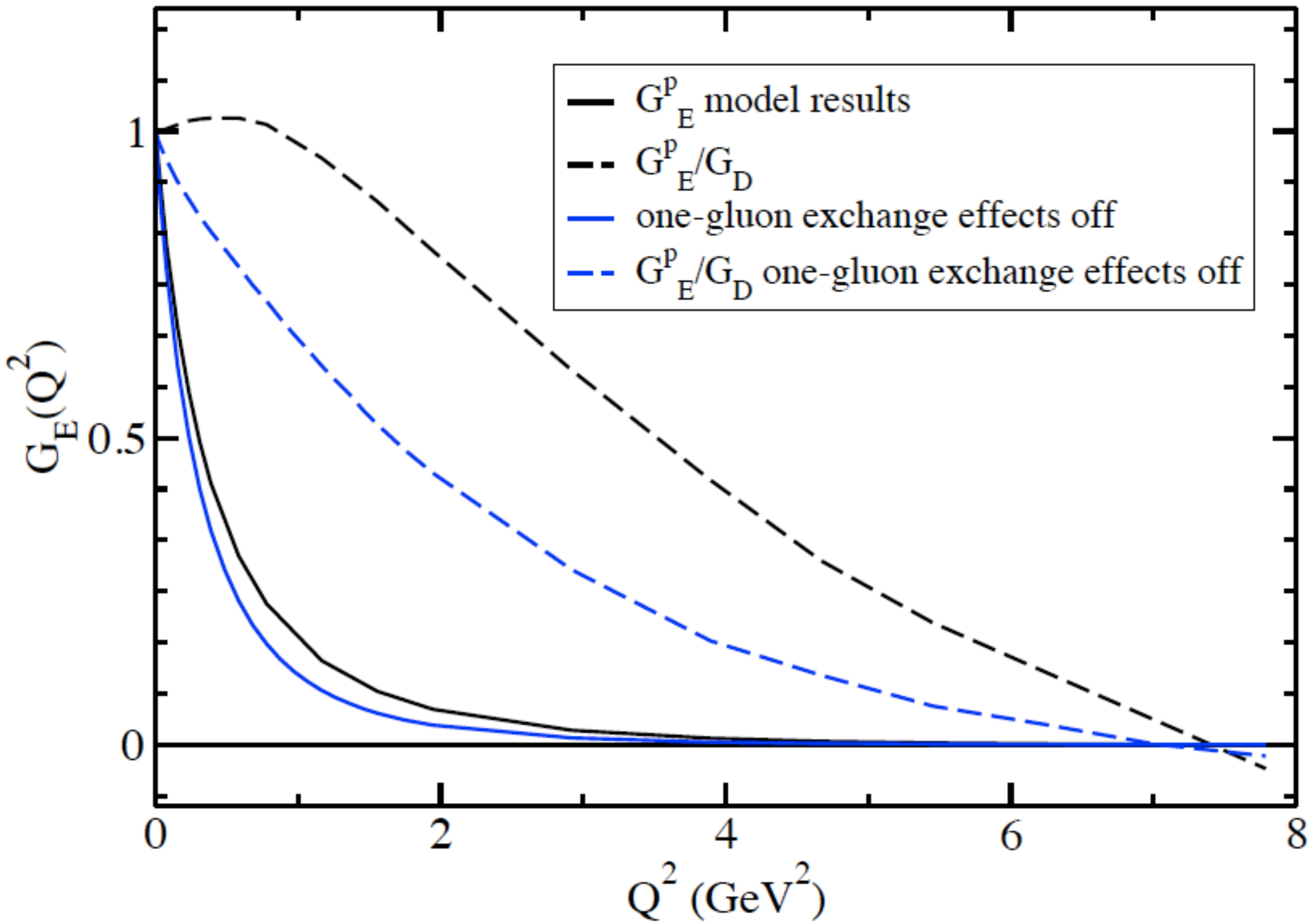
κ_u=0.036, κ_d=-0.125, $F^q_1=(1+Q^2/1.22\ GeV^2)^{-1}$, $F^q_2=(1+Q^2/1.22\ GeV^2)^{-2}$



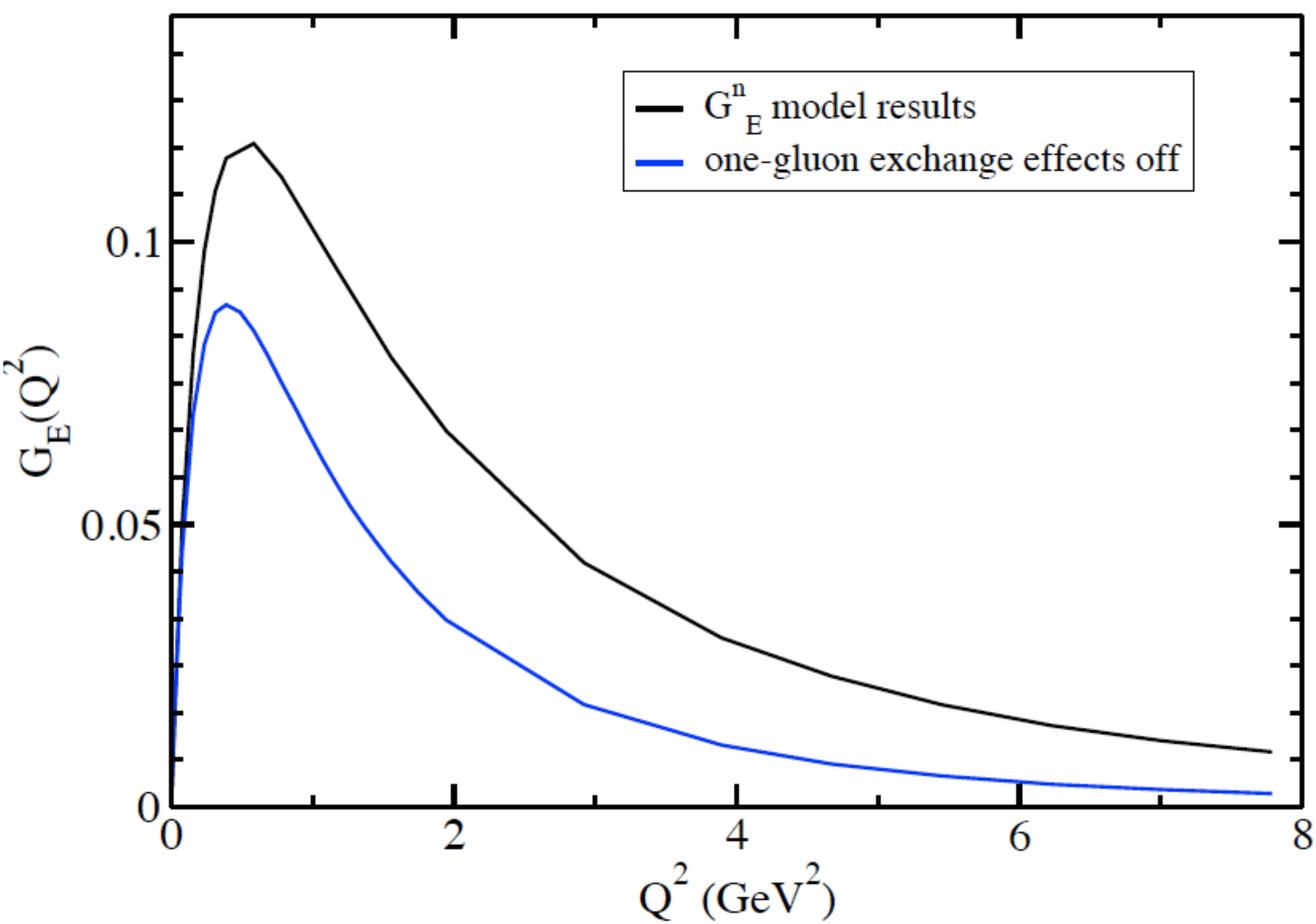
Proton electric form factor



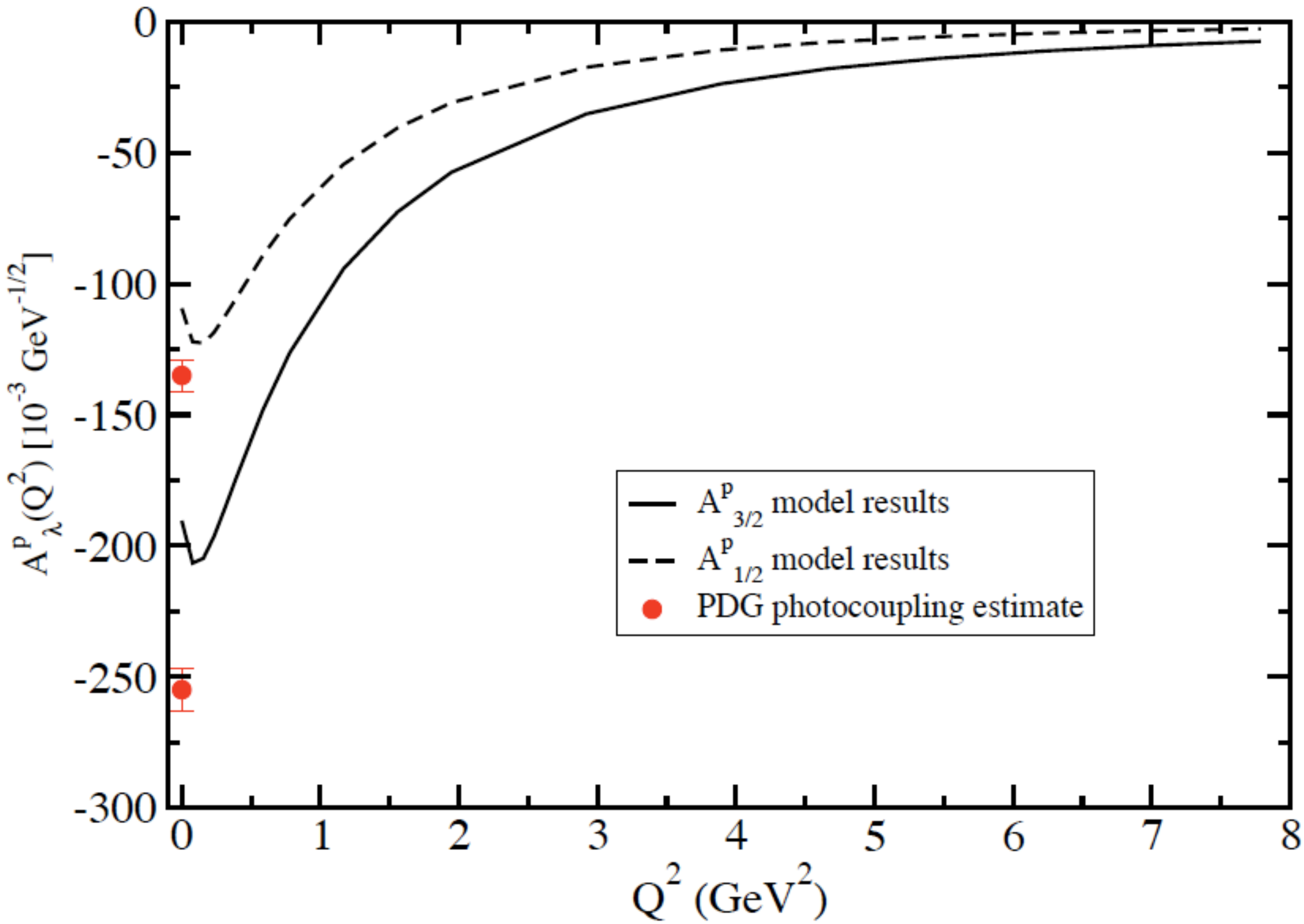
Proton electric form factor



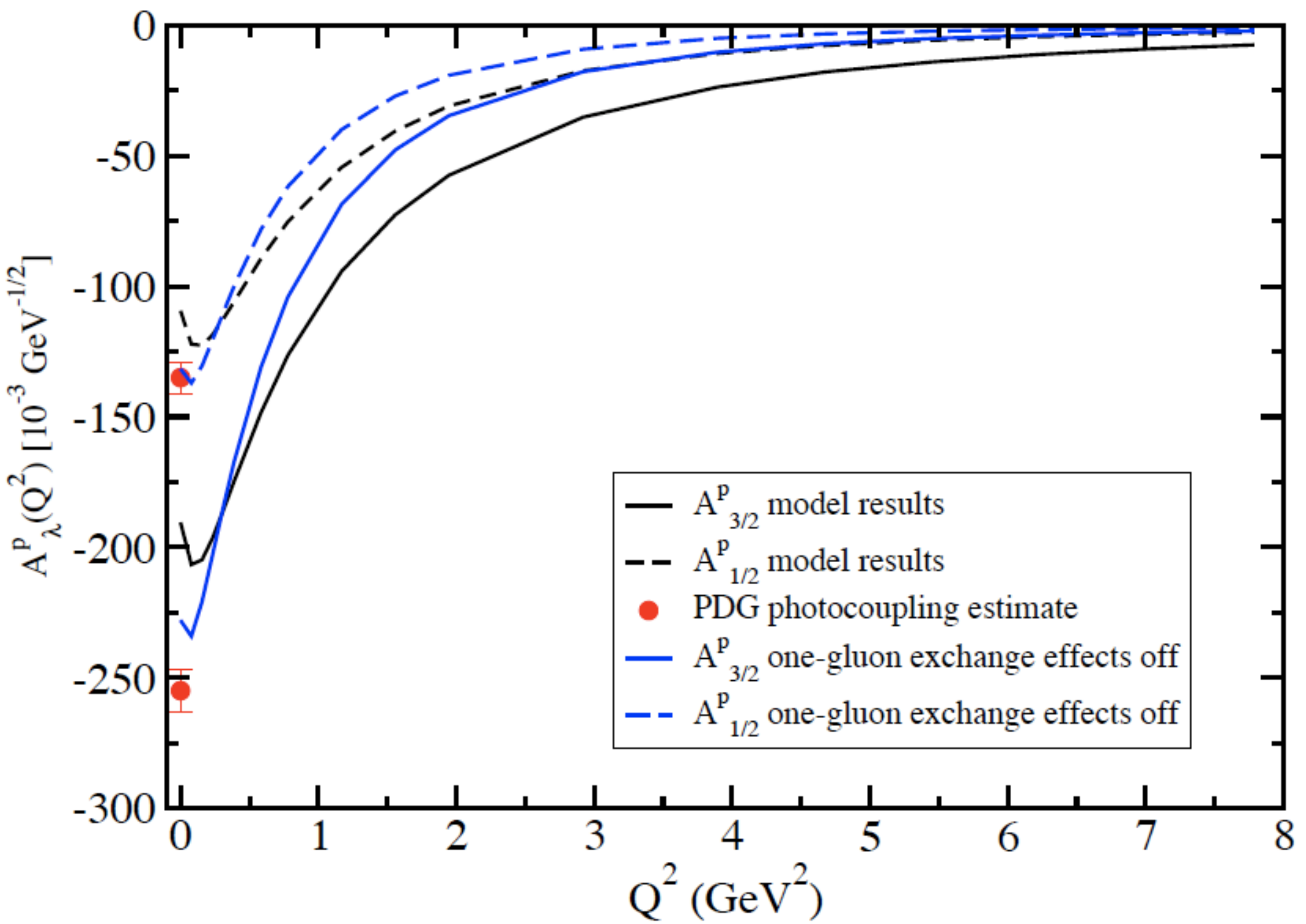
Neutron electric form factor



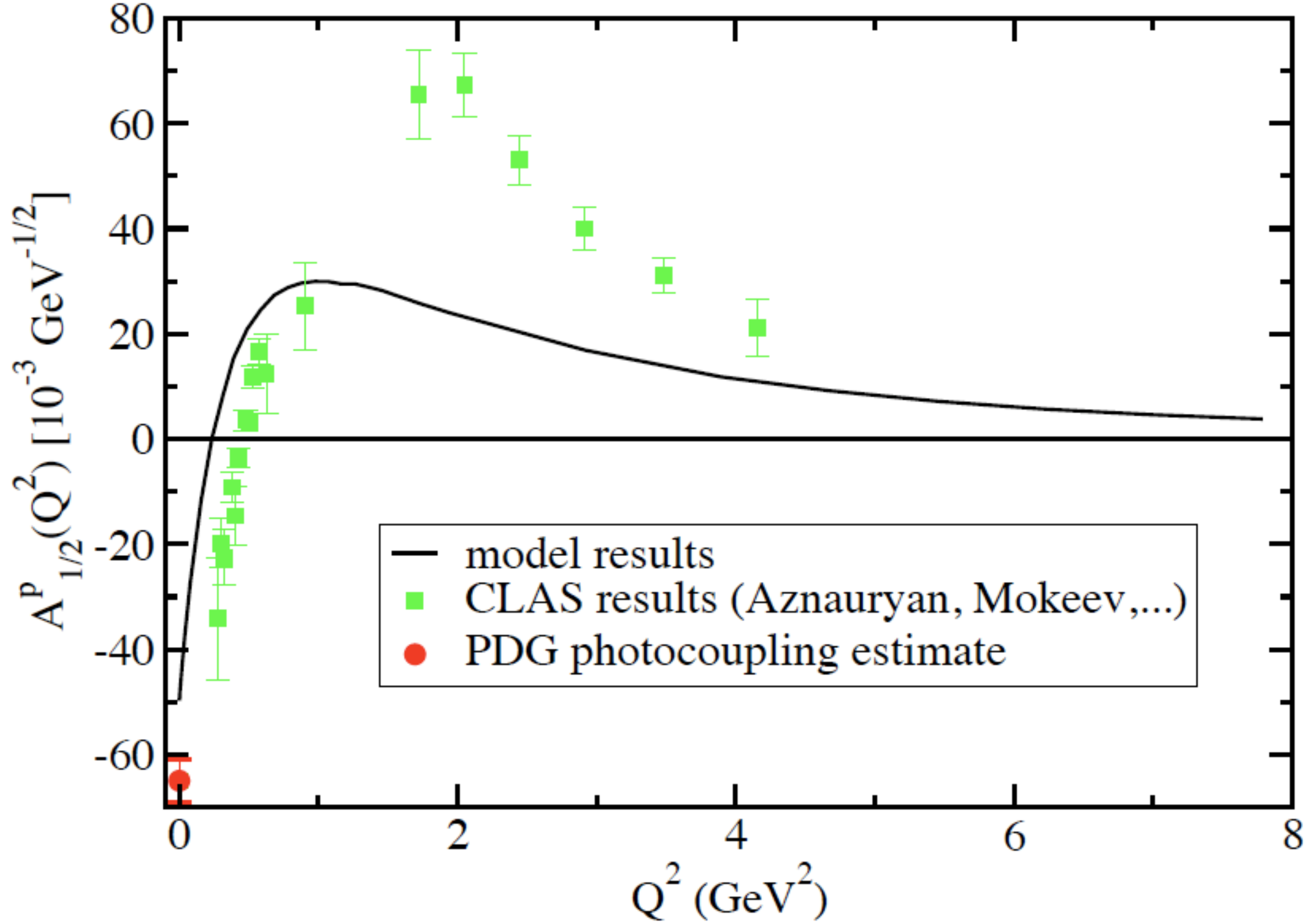
Delta resonance transverse amplitude



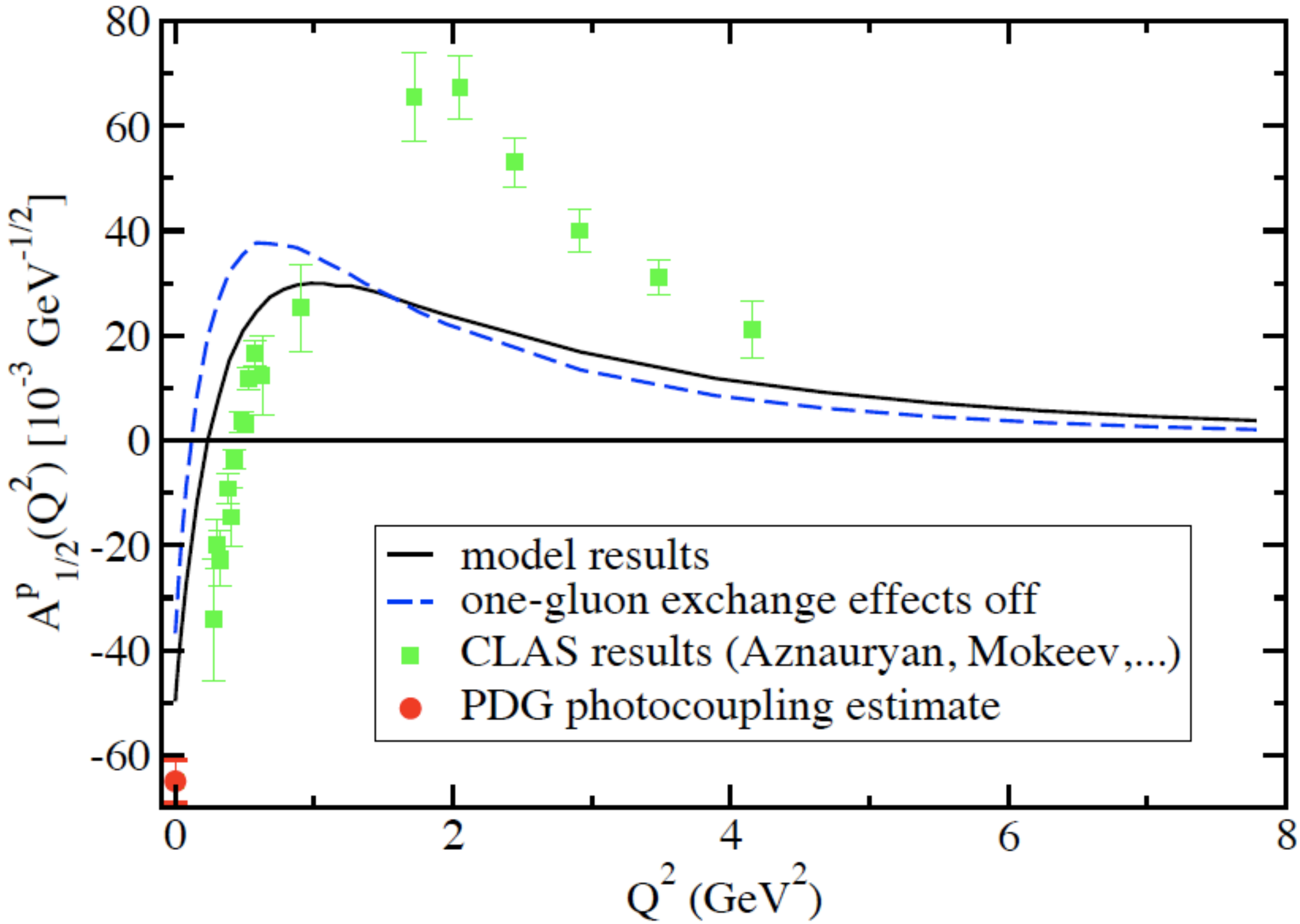
Delta resonance transverse amplitude



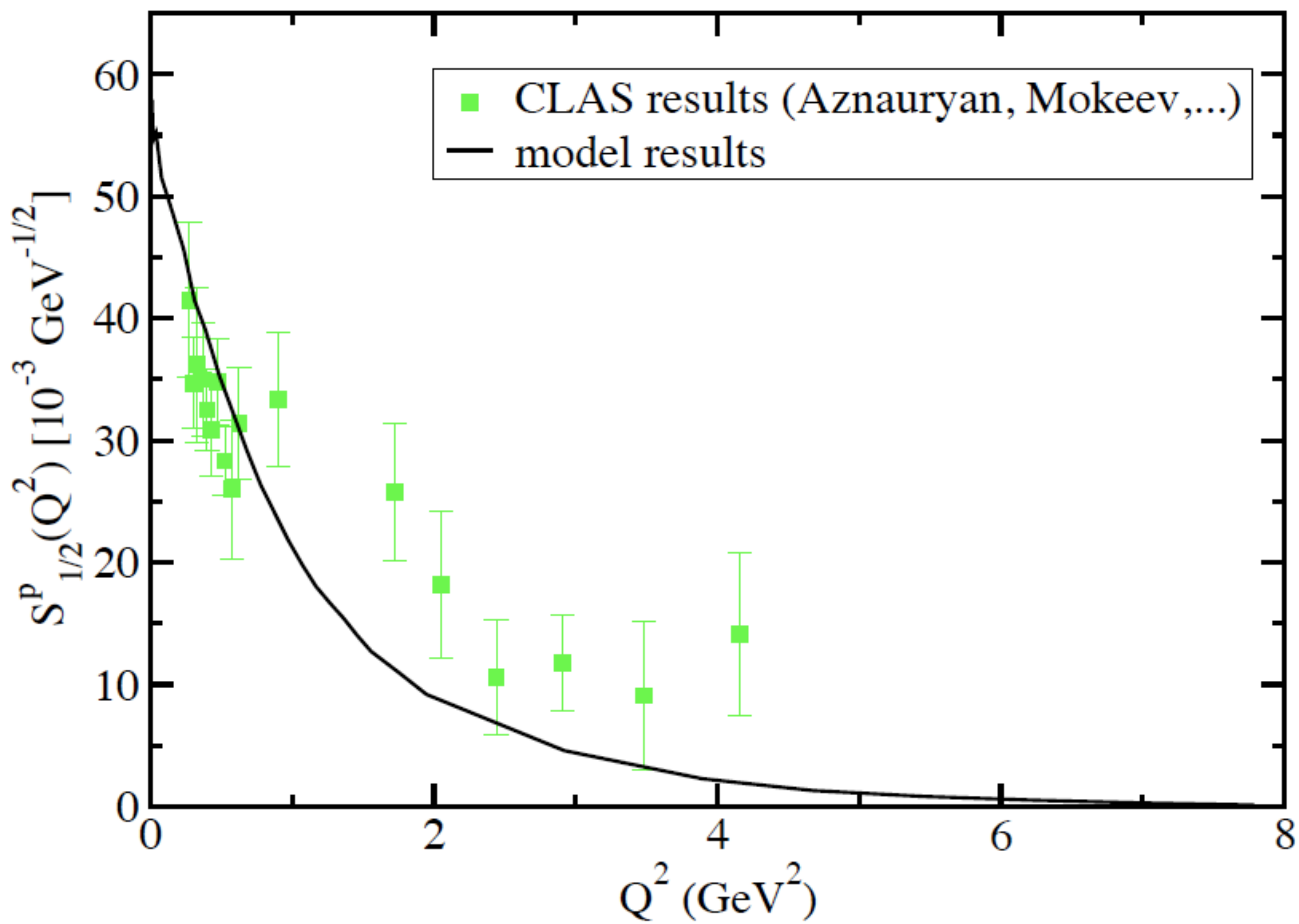
Roper resonance transverse amplitude



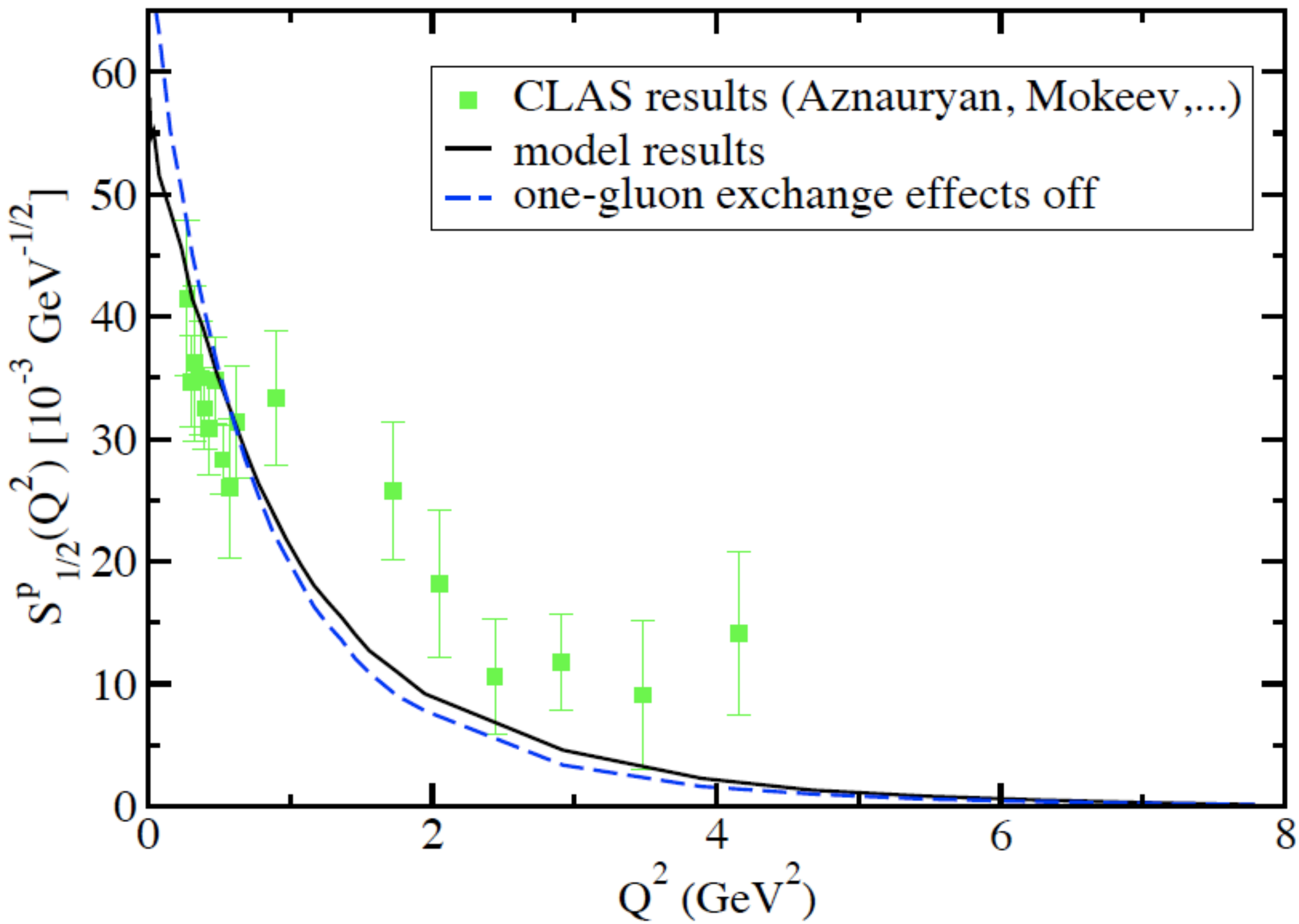
Roper resonance transverse amplitude



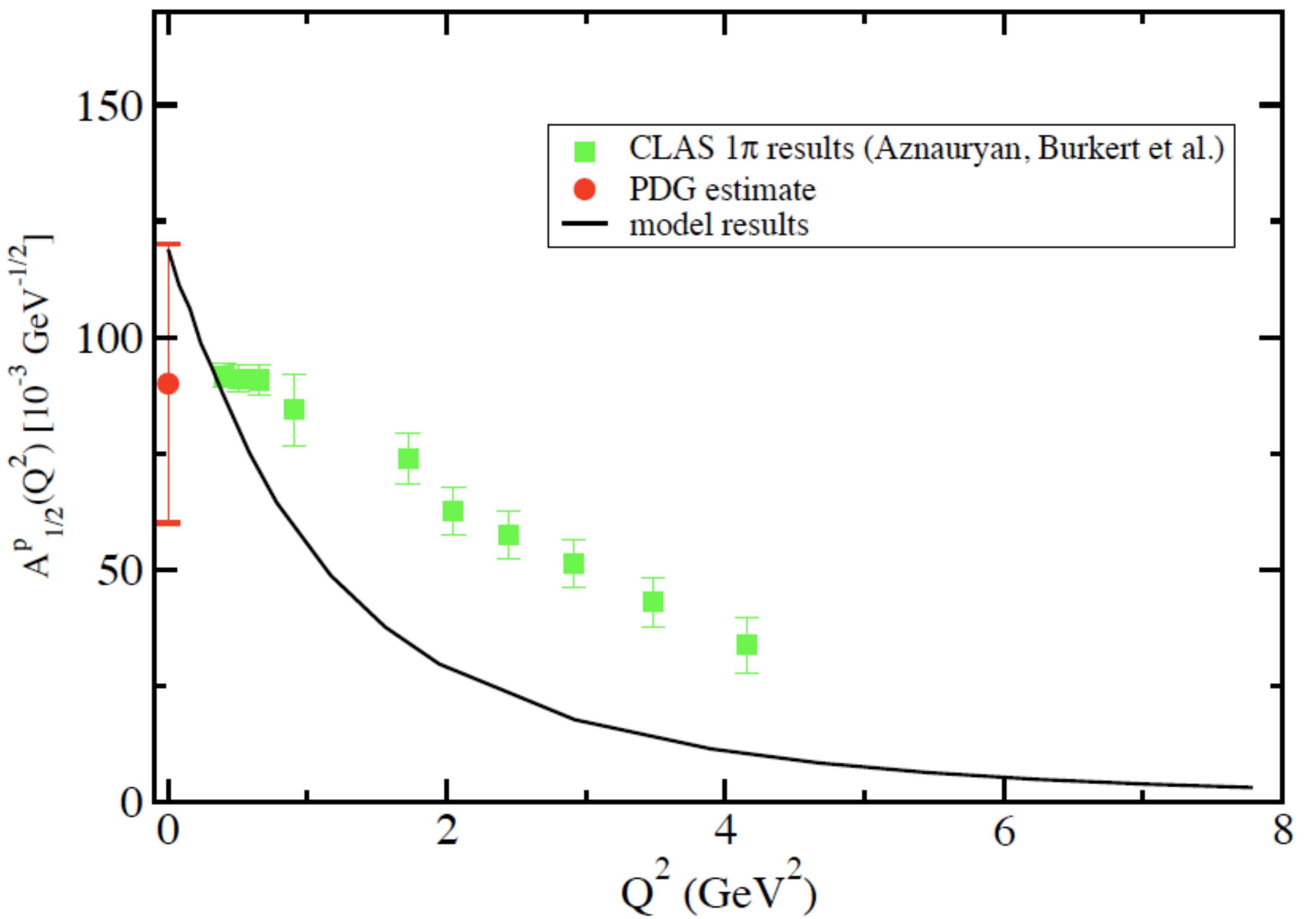
Roper resonance scalar amplitude



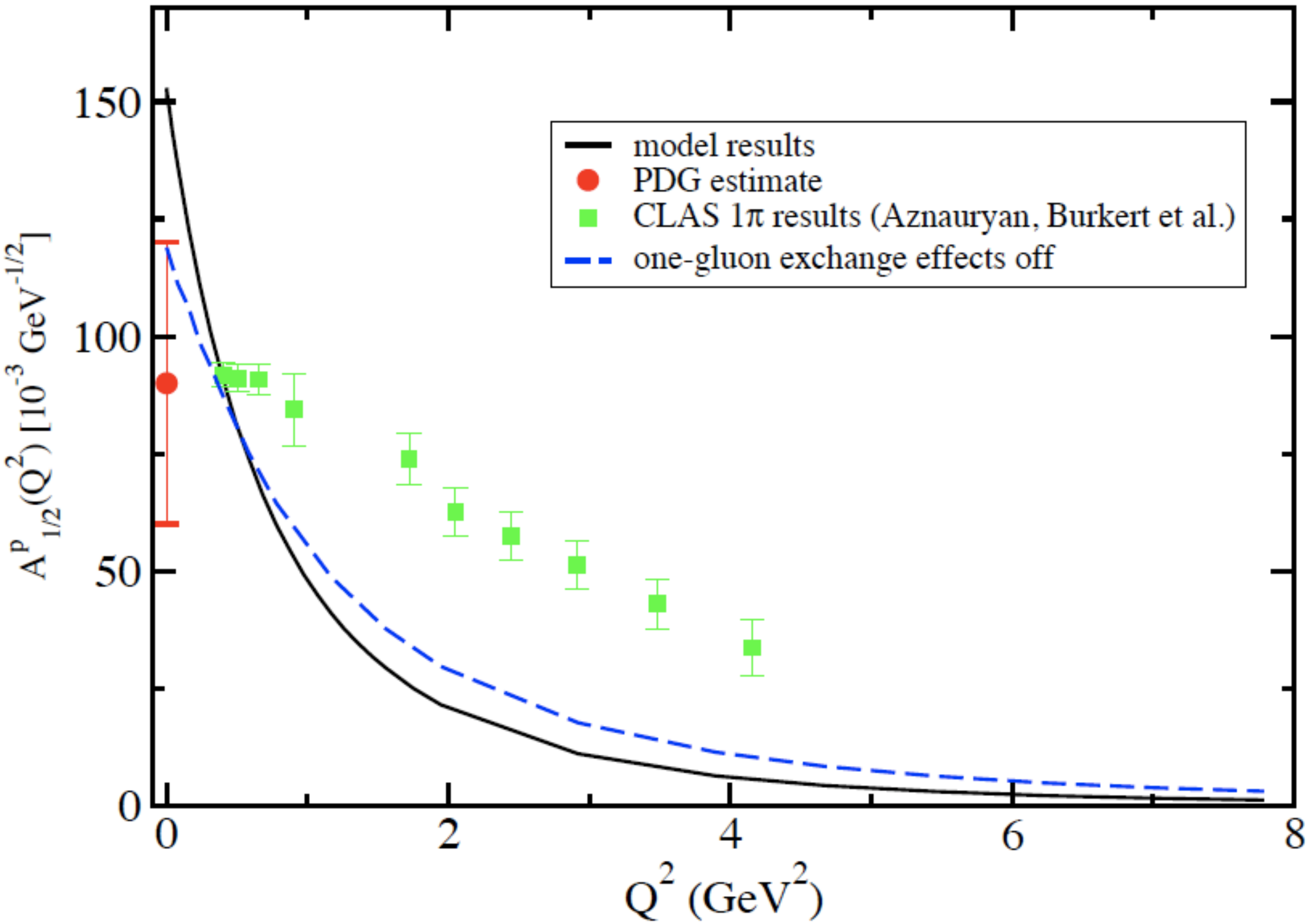
Roper resonance scalar amplitude



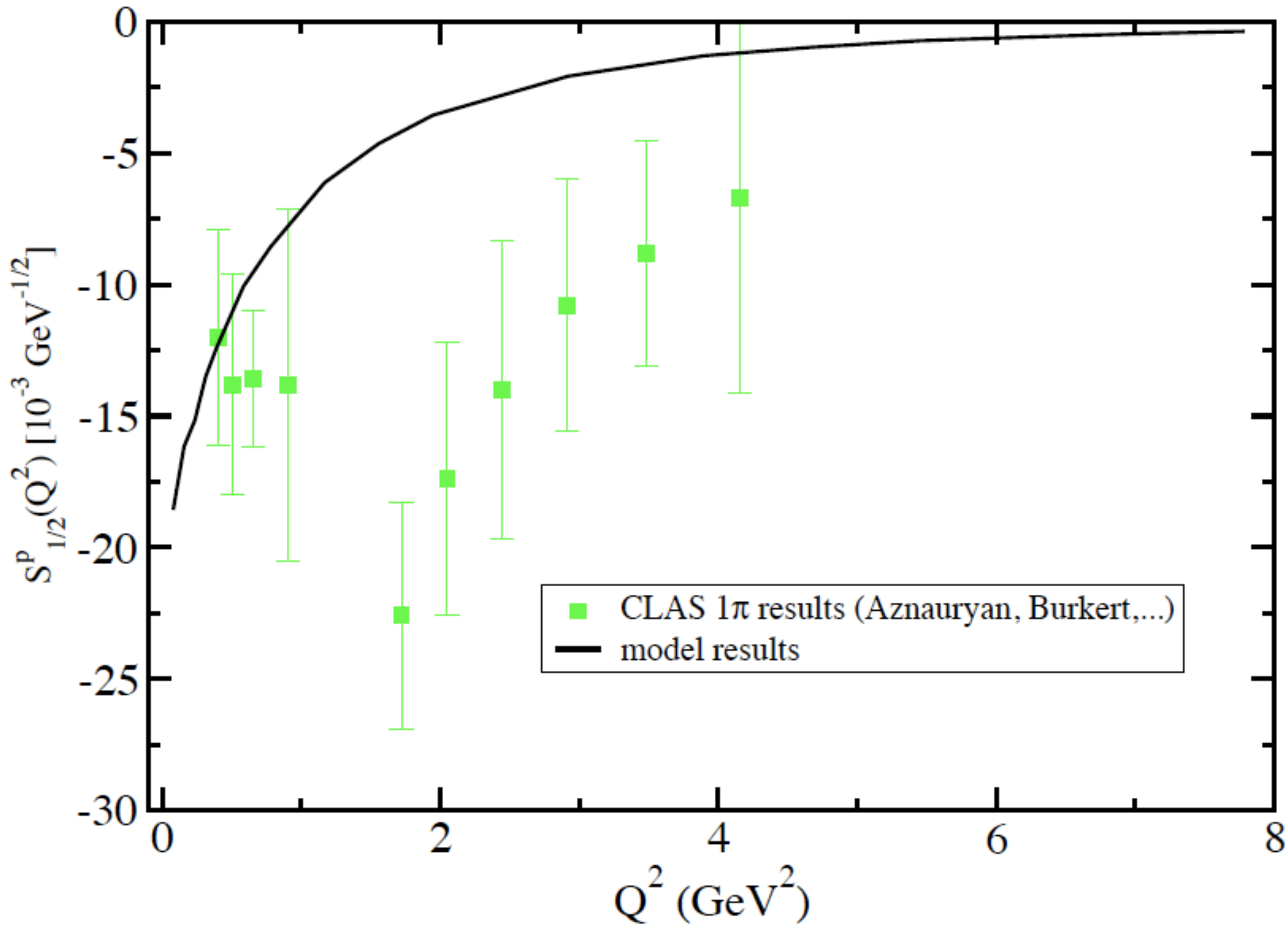
$N(1535)S_{11}$ resonance transverse amplitude



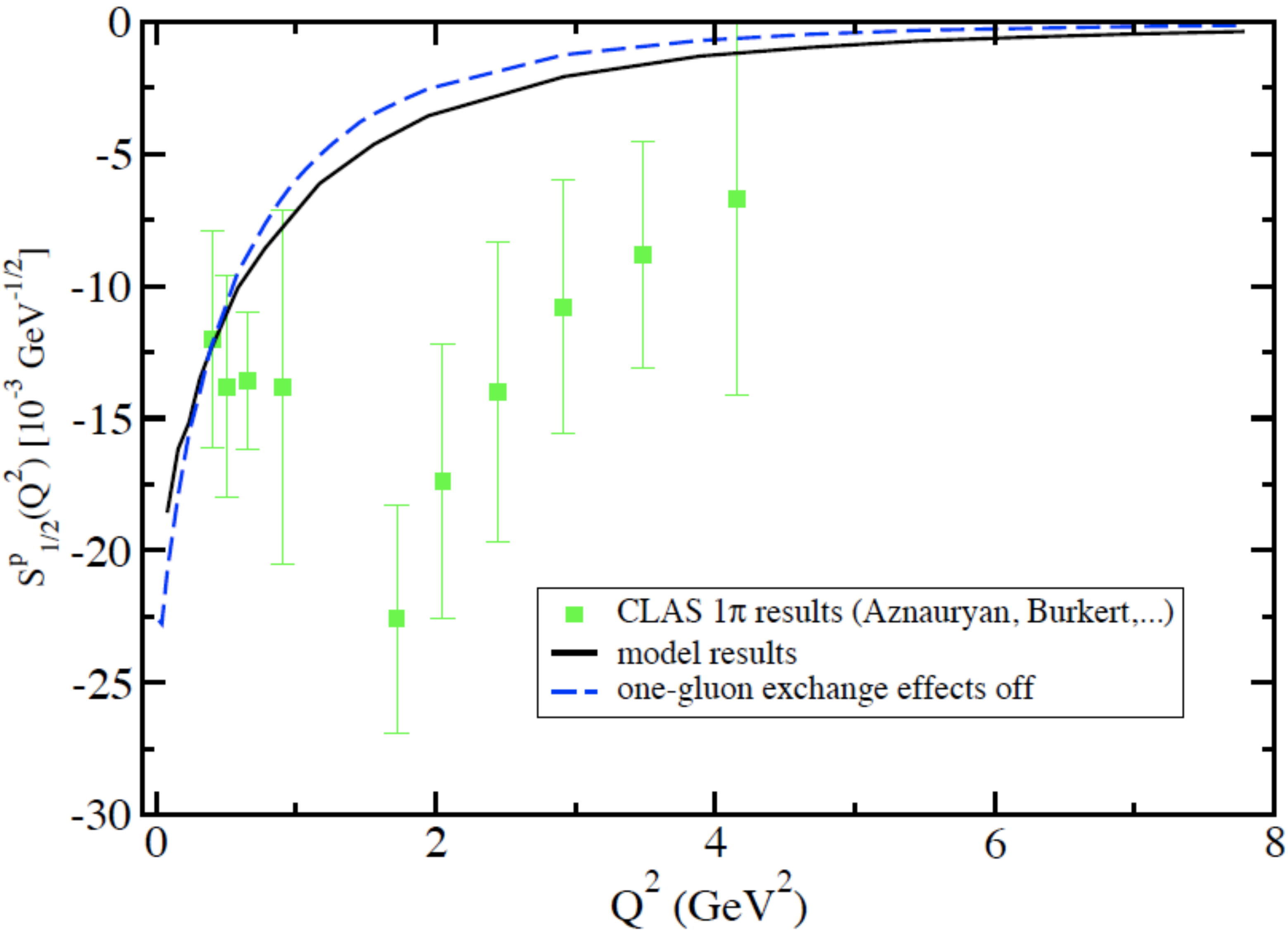
N(1535)S_{11} resonance transverse amplitude



N(1535)S_{11} resonance scalar amplitude



N(1535)S_{11} resonance scalar amplitude



Rotational covariance

- States with higher J
  - Rotations are dynamical in light-front QM
  - It is possible to quantify the violation of rotational covariance by forming a linear combination of light-front spin matrix elements which should be zero
    - E.g. for $\Delta(1232)$ there is one such combination
      - Becomes comparable to $A^p_{3/2}$, $A^p_{1/2}$ only at higher Q^2
      - Calculation of sub-dominant amplitudes (E1+, S1+) believable at Q^2 below roughly 2 GeV^2
    - Non-zero because calculation truncated at one-body currents

Rotational covariance…

- For states with J=5/2 there are three linear combinations which should be zero
  - For $N5/2^+(1680)$ these may not small at 1 GeV^2
- Some authors claim to have a work around for J=1/2
  - Evaluate light-front matrix elements of other components of the EM current, take linear combinations to eliminate matrix elements which must be zero
  - But there is no free lunch for higher J!
    - If use other components of I, don't have minimal set of matrix elements which transform into each other under boosts

Conclusions/Outlook

- Calculation of EM transition form factors for low J using light-front dynamics is reliable
  - We have made a simple fit to nucleon form factors extracted from polarization data
    - Results for nucleon similar to those of Miller
  - We have looked at nucleon, $\Delta(1232)P_{33}$, $N(1440)P_{11}$, $N(1535)S_{11}$
    - (see also Rome group)
    - Effects of configuration mixing (one-gluon exchange, confining potential) are substantial
    - Relativistic effects can be large, e.g. Roper
  - Model can be applied to any state
  - Can estimate uncertainties at higher Q^2 from lack of rotational covariance
- Working on transitions to N* with higher J